

Dimensioning of longitudinal reinforcements in concrete beams with a non-rectangular section and variable height

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Abstract

Lately, structural typologies of agricultural infrastructures have undergone significant changes. These have been especially important in the case of farm buildings where reinforced concrete structures are the most commonly used. In order to make precast concrete items which are interesting because of their cost, it is necessary to optimize their design and implement accurate calculation procedures. This paper presents a methodology to calculate longitudinal reinforcements in reinforced concrete (RC) beams with non-rectangular sections and a variable height along the main direction of the beam. This method has been developed taking into account the interaction diagrams of the different sections in which the beam has been previously split. Checks included considering the contribution of all the reinforcements in a section, including the support reinforcements. For each section, the different combinations of resistant axial force and bending moment were estimated, i.e. their interaction diagram was drawn. Once the axial force of the studied beam was known, the bending moment of each section was calculated. These values were then integrated for the whole beam and the maximum moment diagram was represented. By repeating the whole process for different reinforcements, the optimal reinforcement could be determined for each loading situation in the beam. A software tool has been developed so this methodology can be easily performed and design of the RC beams optimized.

Additional key words: building, interaction diagram, precast reinforced concrete.

Resumen

Dimensionamiento de las armaduras longitudinales en vigas de hormigón con sección no rectangular y canto variable

En los últimos años las tipologías estructurales de las infraestructuras agrarias están sufriendo importantes cambios, especialmente destacables en el caso de los alojamientos ganaderos, en los que cada vez más predominan las estructuras prefabricadas de hormigón armado. Para conseguir productos prefabricados de hormigón que por su coste sean interesantes, es necesario optimizar su diseño y hay que aplicar procedimientos de cálculo que sean precisos. Este trabajo presenta una metodología para calcular las armaduras longitudinales en vigas de hormigón armado, que tengan una sección que pueda ser diferente de la rectangular, y que además presenten una altura variable a lo largo de la directriz de la pieza. Este método es desarrollado a partir de los diagramas de interacción de las distintas secciones en las que previamente se discretiza la viga. Se realizó la comprobación teniendo en cuenta la aportación de todas las armaduras que intervienen en la sección, incluidas las armaduras de piel, determinando para cada una de las secciones las diferentes combinaciones de esfuerzo axial y momento flector que resistiría la sección, es decir, dibujando su diagrama de interacción. Conociendo para la viga en estudio el esfuerzo axial al que estaría sometida, se obtuvo el momento flector de cada sección, integrándolos para toda la viga y representando de esta manera el diagrama de momentos últimos. Realizando todo el proceso para diferentes armados, se determinó el armado óptimo para cada situación de cargas en la viga. Esta metodología de cálculo se ha implementado en una aplicación informática, y de esta manera se puede utilizar para optimizar el diseño de las vigas de hormigón armado.

Palabras clave adicionales: construcción, diagrama de interacción, hormigón armado prefabricado.

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Introduction

In recent years, structural typologies of agricultural infrastructures have undergone important changes, not only due to market pressures, but also because of the need to optimise costs, assembly time, and structural aspects, both from an aesthetic and a versatility point of view. This is especially important in the case of farm buildings, where new materials other than structural steel, although resistant to corrosion caused by animal droppings, have had to be introduced, explaining the interest in reinforced concrete (RC) structures. Despite the fact that these structures have always been more expensive than metallic ones, lately, the cost of steel has increased and, nowadays, concrete structures can have a similar price. Moreover, because of increasing use of more resistant concretes and the need to cover large-span structures, people are looking for new solutions. This is where precast concrete structures are more competitive, due to their high versatility, low cost, reduced maintenance and easy assembly.

Precast RC structures are being increasingly introduced in the farm building sector, especially in frames and enclosures. They make a real difference when time of assembly and quality of implementation are considered.

These building elements should not only be more cost-competitive than traditional ones, but their design should also be optimised to suit their intended assembly location.

Companies have developed a series of precast RC elements with different designs, sections and heights, which can be better adjusted to their design requirements. To mention just a few, C channel sections, double T sections, simple T sections, L angular sections, etc. have been developed. However, there is still not enough information available about the behaviour and reinforcing characteristics of these elements.

In most cases, the beam section is a fixed parameter, defined by the need to have precast moulds at their disposal, for their construction. For this reason, it is necessary to implement a calculus method to define the most suitable reinforcement. This will take into consideration a series of external stress conditions for a particular structural typology, which will be defined by the mould design (mould removal slope, symmetric sections, necessary widths, etc.).

For this purpose, the research group is collaborating with the precast concrete company *Prefabricados Madrigueras S.L.*, in order to characterize and improve their products.

Most of the current literature related to the design of RC sections, focuses on reinforced rectangular (or T at the most) sections with a single tensile reinforcing area and a single compression reinforcing area. It is, therefore, necessary to develop a methodology to perform an exhaustive calculus of the sections whatever their shape, considering the contribution of numerous reinforcing areas, both in tension and compression. Needless to say, Spanish EHE (1998) and Eurocode 2 (AENOR, 1998) standards must also be taken into account.

The methodology developed has been based on obtaining the interaction diagrams of the different sections into which the beams of variable height had been previously divided. Although this method has already been mentioned by some authors (Calavera, 1999; Jiménez *et al.*, 2000; García, 2001), there is no account in the literature describing a similar situation, or software tools capable of calculating the dimensions of concrete beams of variable height, whatever their section.

There are several studies about stress-strain relations in RC beams (Careira and Chu, 1986; Williams, 1986; Prakhya and Morley, 1990; Kaklauskas *et al.*, 1997; Kaklauskas and Ghaboussi, 2001). This study, however, aims to develop a method to calculate longitudinal reinforcing in concrete beams (whatever their sections), taking into account the resistant contribution of each reinforcement, regardless of its distribution or position. This method could be generalized, so that beams of variable height could be estimated, with the help of a software tool.

Methodology

A mathematical tool was developed using Visual Basic programming language, in order to carry out the process. This tool integrates the following aspects:

The beam and its reinforcing design

Firstly, design of the structural element is defined: beam length, section shape, reinforcing, etc. (Fig. 1).

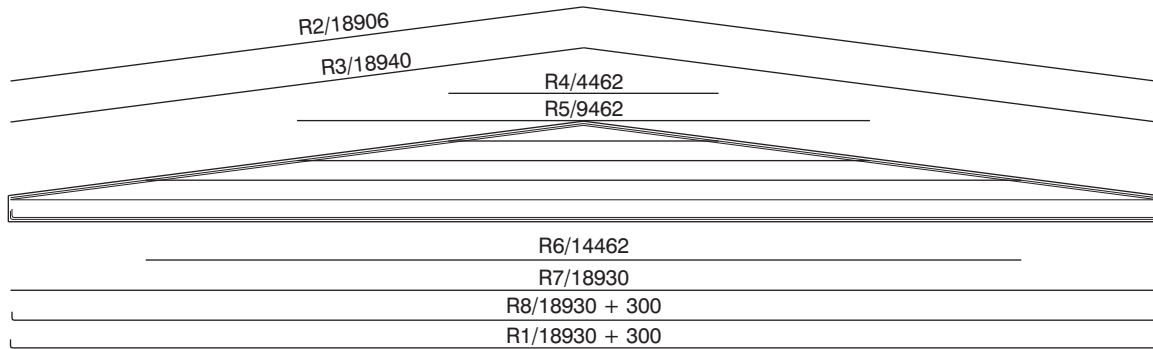


Figure 1. Example of a beam reinforcing diagram (number of reinforcing and its length in mm).

Calculation of the beam internal stresses

Internal stress diagrams are calculated using the external loads applied to the beam: shear (V) and axial (N) stress, and bending moments (M). Both axial stress and bending moments will be supported by longitudinal reinforcement, and shear stress will be supported by shear reinforcement (although this has not been considered in the present study).

Splitting the beam with variable height into a finite number of parts

The resistant section and the reinforcement of the beam of variable height depend on the section selected. The beam is divided into a finite number of parts (Fig. 2) and characterized by the central section of each one (Fig. 3). The beam is, therefore, considered to be continuous, by characterizing and fixing the different parts into which it has been divided.

Drawing the interaction diagrams for the different sections

A geometric characterization is provided for each section, and the starting parameters for the calculation are defined. The process is based on drawing a diagram for each section (with a non-rectangular shape) and on considering all the reinforcements (Fig. 3).

Each section, subject to a central bending, will be requested by two normal stresses (N, M). The aim of the calculation is to determine the different pairs of axial - maximum moments the section is able to resist. This way the ultimate limit state of collapse due to normal stresses can be checked (EHE, 1998). Each section subjected to normal stresses can reach the collapse limit state because of:

- An excessive steel plastic strain: this is avoided by establishing 10‰ as the limiting value.
- Concrete being crushed under bending: the limit of compression due to bending is fixed at 3.5‰.

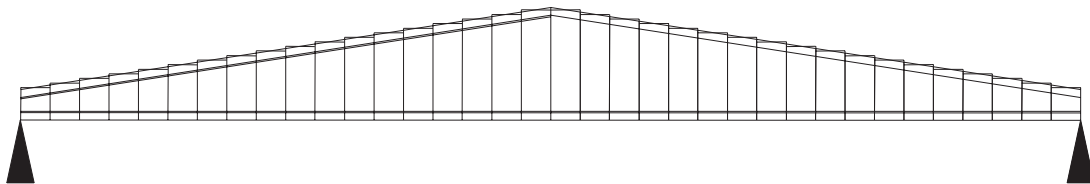


Figure 2. Splitting the beam into a finite number of parts.

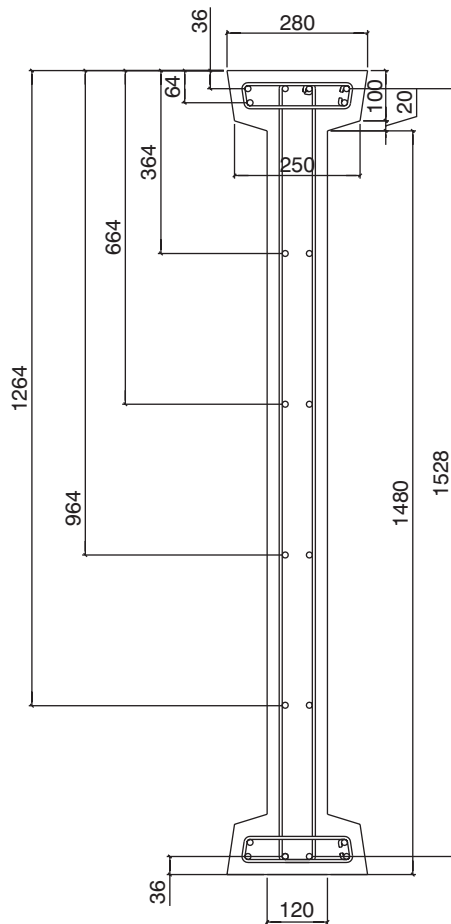


Figure 3. Generic section of a beam, showing both reinforcement and size (in mm).

- Concrete being crushed under compression: both under simple and compound compression; 2‰ is established as the maximum concrete strain.

There are a series of collapse situations that correspond to the different normal stress conditions continuously covered, from simple tension to simple compression. Strain in two of the section fibres is known for each situation.

Longitudinal reinforcement undergoes the same deflection as the surrounding concrete, under the stresses the section is subjected to. The Bernoulli hypothesis is assumed. This hypothesis states that normal sections remain plane after deformation (this law is valid whenever the length of the beam between two points of null moment is greater than twice the beam total height).

Thus, once the strain in two fibres of the section and their lineal variation law is known, the strain is defined in all the fibres. Resistant stress is then determined by means of both the steel and concrete strain-stress diagram models, as indicated in the Spanish EHE standard (1998) (Fig. 4). According to this, the maximum strain resisted by the reinforcements is the «steel design strength, f_{yd} » and 85% of the «concrete design strength, f_{cd} ».

Finally, the calculation is based on studying the equilibrium of all the forces and moments on the section, which takes into consideration both the yield strength of materials and the internal stress. When a section has reinforcements in just two positions, dimensioning of reinforcements is done according to internal stress. However, if the section has more than two reinforcement positions, as is the case of large-span and variable section height beams, yield strength of the section is checked in the existing reinforcements (interaction diagrams).

In order to do so, the «deformation domains» or «pivot diagrams» are applied (Fig. 5), i.e. a graph representing the section limit strains depending on the neutral axis (X) position, for each of the standard stress conditions (Calavera, 1999; Jiménez *et al.*, 2000; García, 2001).

The deformation domain (1,2,..., 5) is known for each position of the neutral axis (X), and can be used to deduce the limit strain of the most tensioned reinforcement (ϵ_s) and the most compressed concrete fibre (σ_c) (Fig. 5). Taking the Bernoulli hypothesis into consideration, the strain suffered by each of the reinforcements, is obtained by linear interpolation for each reinforcement position.

Thus, taking the most compressed fibre as a reference, d is the «effective depth» of the cross-section (or the position of the most tensioned reinforcement), X is the neutral axis position, d_i is the different A_i reinforcement position, ϵ_c is the extreme compression fibre strain, and ϵ_{si} is each reinforcement strain. The following ratio occurs in domain 2:

$$\frac{10}{d - X} = \frac{\epsilon_c}{-X} = \frac{\epsilon_{si}}{d_i - X}$$

Compressions are negative (−) and elongations positive (+).

In domain 3, the following ratio will occur:

$$\frac{3.5}{X} = \frac{3.5 + \epsilon_{si}}{d_i}$$

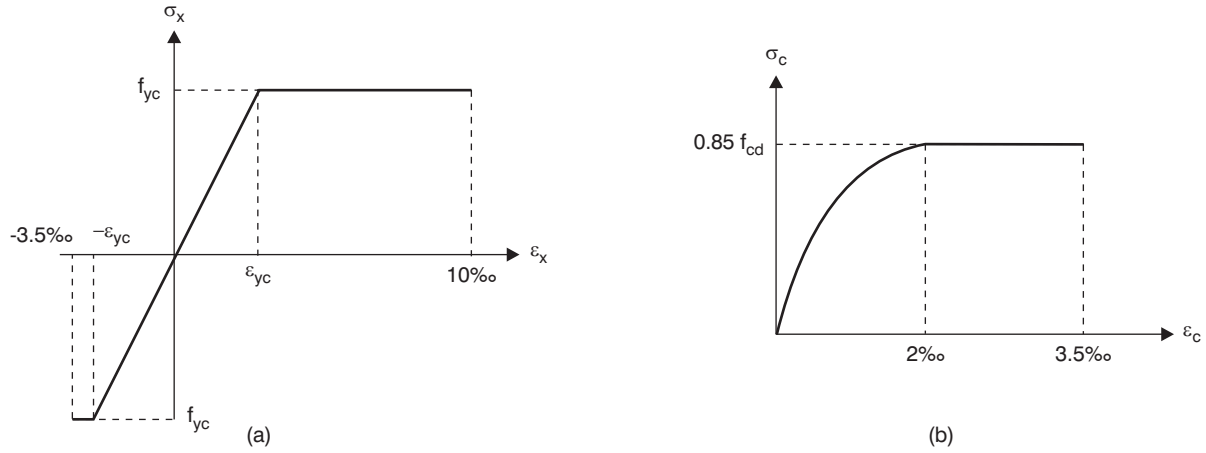


Figure 4. Stress-strain relations for both steel (a) and concrete (b).

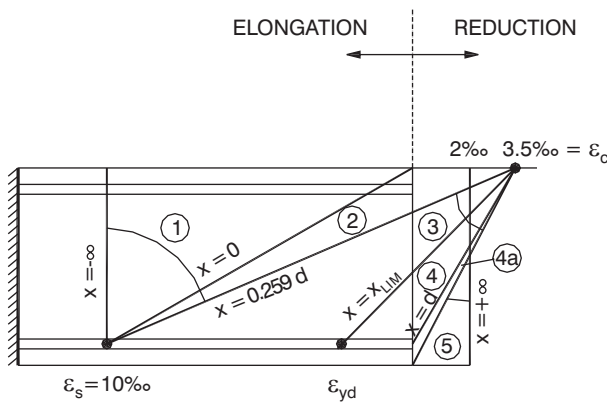


Figure 5. Strain domain diagram.

Once the strain is known, each reinforcement resistant stress is determined through the steel strain-stress diagrams (Fig. 4), depending on the reinforcement working on an elastic or plastic regime.

The resistant force (F_i) is calculated using each reinforcement resistant stress (σ_i), with an area of A_i , by the following formula (Fig. 6):

$$F_i = \sigma_i \cdot A_i$$

Identically, once both the concrete design strength within the compression block and the neutral axis position are known, the concrete resistant contribution can be determined. In order to do so, the parabola-rectangle diagram or the equivalent rectangular one (Calavera, 1999; Jiménez *et al.*, 2000) can be used.

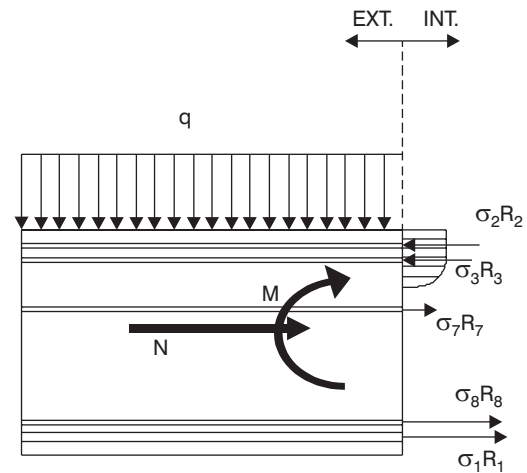


Figure 6. Internal resistance of a reinforced concrete section.

Finally, equilibrium equations are applied to the section ($\Sigma M = 0$; $\Sigma FN = 0$) and the normal resistant stresses (axial and bending moment) are calculated.

By simply applying the process again for the different positions of the neutral axis (from $-\infty$ to $+\infty$), resistant normal forces (axial and moment) are calculated in each section, and represented on a diagram (Fig. 7). This diagram, known as the «interaction diagram», represents a curve that shows maximum moment values for each axial value. As a result, all of the internal points can be considered as acceptable for the cross-section studied, and the external points will be assumed to represent non-resisted stresses for the section.

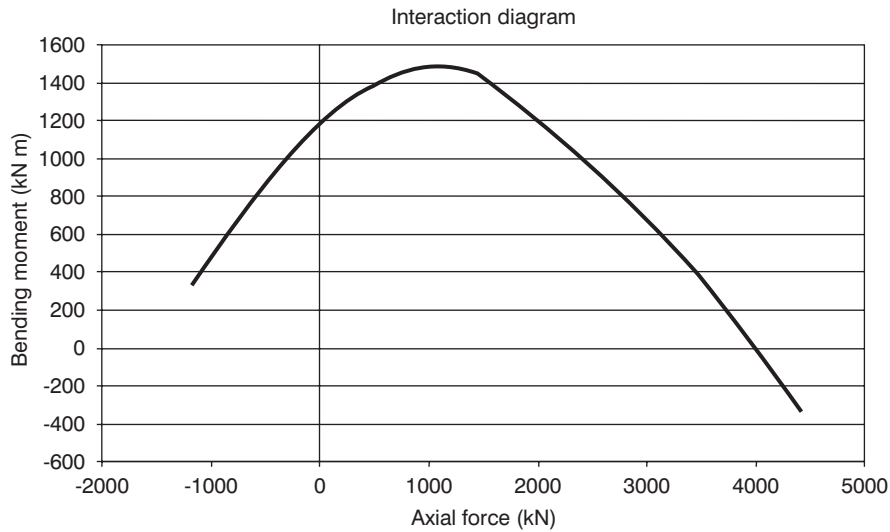


Figure 7. Interaction diagram of a beam section.

Drawing of the beam moment diagram

The shape section has been considered as a fixed parameter because, in the case of precast concrete, it depends on the mould. Consequently, both reinforcing and concrete characteristics determine the section resistance. The bending moment resistance is calculated for each section, according to the axial force value. Finally, data are displayed on a graph.

Comparison with the moment diagram and the definitive reinforcement

Finally, internal bending moments (Fig. 3) are compared graphically with resistant moments, taking into consideration each type of reinforcement (Fig. 8). The reinforcing proposal recommendation should then be studied and the decision should be made about whether the beam design can be optimised.

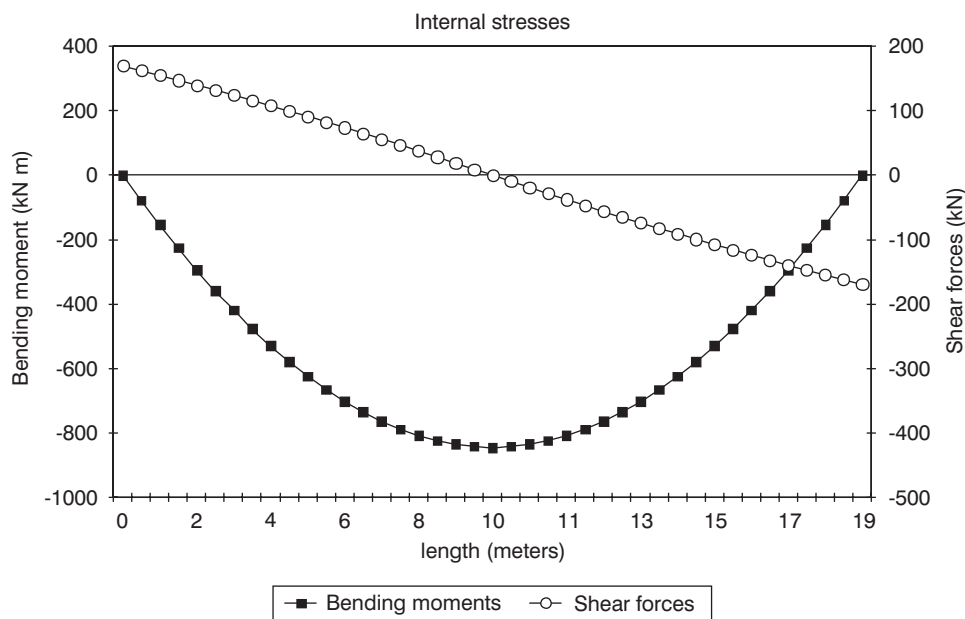


Figure 8. Internal stresses (shear forces and bending moments) in a 19 m length beam with a distributed external load of 7.5 kN m^{-1}

Checking

It is important to take into account that this study focuses on calculating the beam longitudinal reinforcement, and should be completed by estimating shear reinforcement, deflection, anchorage, etc.

Process application

As an example, the calculation process is applied to a beam of a farm building with a 19 m span and a distributed load of 7.5 kN m^{-1} .

- *Beam design and its reinforcing.* A by-supported beam with the following characteristics has been considered: 19 m length, 400 mm initial section height, «I» section, upper slope of 12%, 120 mm web thickness, and eight different positions of longitudinal reinforcement (Fig. 1). The aim of this example is to evaluate three reinforcements and decide whether they are valid, as summarized in Table 1. Longitudinal reinforcements made of steel B-400S ($f_{yk} = 400 \text{ N mm}^{-2}$) and HA-25 RC ($f_{ck} = 25 \text{ N mm}^{-2}$) have been used.

- *Calculation of the beam internal stresses.* Internal stress diagrams are calculated using the external loads applied to the beam (7.5 kN m^{-1} plus its dead load). In this example, the by-supported beam was only affected by shear forces (used to calculate the shear reinforcement) and bending moments (used to estimate longitudinal reinforcement) (Fig. 8).

- *Dividing a variable height beam into a finite number of parts.* Forty sections have been studied, considering that this number would give sufficient accuracy for the cases studied (Fig. 2).

- *Drawing of the interaction diagrams for each section.* Numerous positions along the neutral axis have

been considered for each section, with increments of 0.02 m. In this way, the whole of the deformation domains are taken into account (Fig. 5). As explained in the above section, the Methodology, the limit strain of both the most tensioned reinforcement and the most compressed concrete fibre are calculated for each neutral axis position. Moreover, a linear interpolation is applied to estimate the strain each reinforcement undergoes. Fig. 9 shows an example of the calculation of the elongation suffered by the different reinforcements located in the central section of the beam, when the neutral axis is at $X = 0.74 \text{ m}$, i.e. at domain 3. The results are as follows: $\varepsilon_c = -3.5\%$; $\varepsilon_1 = 4.06\%$; $\varepsilon_2 = -3.33\%$; $\varepsilon_3 = -3.20\%$; $\varepsilon_4 = -1.61\%$; $\varepsilon_5 = -0.19\%$; $\varepsilon_6 = 1.23\%$; $\varepsilon_7 = 2.64\%$; $\varepsilon_8 = 3.93\%$.

When the strain is known, the stress parameter (σ_i) of each reinforcement can be determined from the steel strain-stress diagrams (Fig. 4). The results of these determinations (all expressed in N mm^{-2}) are: $\sigma_1 = 348$; $\sigma_2 = -348$; $\sigma_3 = -348$; $\sigma_4 = -323$; $\sigma_5 = -39$; $\sigma_6 = 245$; $\sigma_7 = 348$; $\sigma_8 = 348$. Considering the resistant stress of each reinforcement, their resistant force (F_i) is calculated using $F_i = \sigma_i \cdot A_i$. The results (expressed in kN) are as follows: $F_1 = 560.023$; $F_2 = -157.352$; $F_3 = -78.676$; $F_4 = -72.963$; $F_5 = -8.773$; $F_6 = 55.417$; $F_7 = 78.676$; $F_8 = 78.676$. The resulting force of the concrete compression block is also estimated, and corresponds to 1.23 kN in this example.

Finally, the equilibrium equations are applied to the section ($\Sigma M = 0$; $\Sigma FN = 0$) and the normal stresses, axial ($N_{\max}$) and bending moment ($M_{\max}$), for each neutral axis position are calculated. For this section and $X = 0.74 \text{ m}$, the results are: $M_{\max} = 1454 \text{ kN m}$ and $N_{\max} = 775 \text{ kN}$.

The same process is repeated for the different positions of the neutral axis and normal stresses are calculated (axial and bending moment) in each section. These values are then displayed graphically in the

Table 1. Values of the reinforcements considered in the exemplified beam

	Reinforcement 1	Reinforcement 2	Reinforcement 3
R1	2Ø25+2Ø12	2Ø25+2Ø20	2Ø25+2Ø20
R2	2Ø12+2Ø12	2Ø25+2Ø12	2Ø12+2Ø12
R3 to R8	2Ø12	2Ø12	2Ø12
Longitudinal reinforcement total weight (kg)	377	541	435

[illegible]

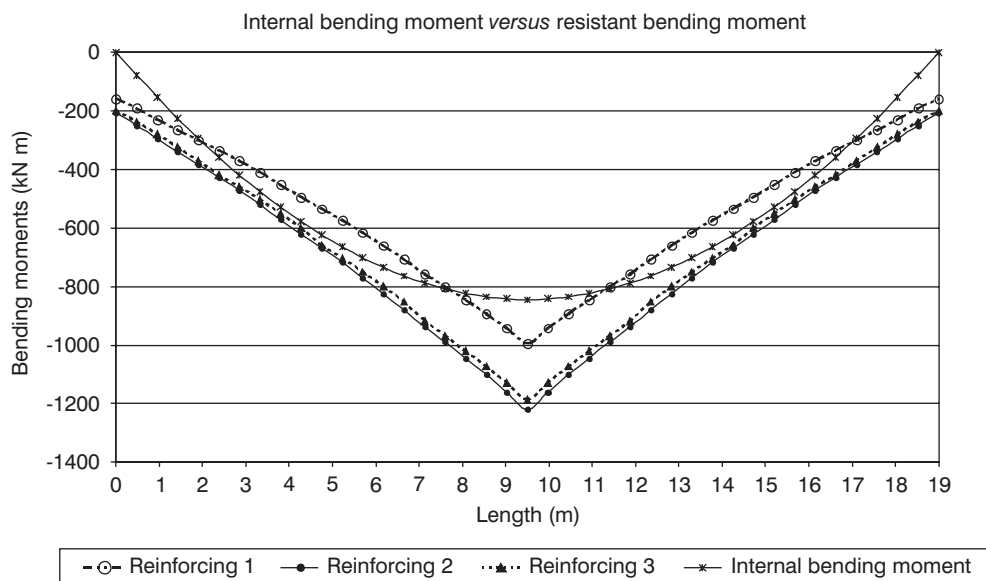
Table 3. Forces due to central section reinforcements of the example beam and normal stresses (N, M) for each of the neutral axis positions.

Depth X (mm)	Forces (kN)									N(kN)	M (kN m)
	R1* σ 1	R2* σ 2	R3* σ 3	R4* σ 4	R5* σ 5	R6* σ 6	R7* σ 7	R8* σ 8	Concrete		
-10000	560.0	157.4	78.7	78.7	78.7	78.7	78.7	78.7	0	-1189	325
-80	560.0	62.5	38.8	78.7	78.7	78.7	78.7	78.7	0	-1055	429
0	560.0	20.4	18.1	78.7	78.7	78.7	78.7	78.7	0	-992	477
140	560.0	-64.5	-23.6	78.7	78.7	78.7	78.7	78.7	-300	-565	804
260	560.0	-151.4	-66.2	47.0	78.7	78.7	78.7	78.7	-577	-127	1111
400	560.0	-157.4	-78.7	-0.4	78.7	78.7	78.7	78.7	-768	130	1251
500	560.0	-157.4	-78.7	-32.0	63.0	78.7	78.7	78.7	-904	313	1328
640	560.0	-157.4	-78.7	-59.6	14.6	78.7	78.7	78.7	-1094	579	1414
860	560.0	-157.4	-78.7	-78.7	-29.6	25.6	78.7	78.7	-1393	995	1483
1060	560.0	-157.4	-78.7	-78.7	-53.9	-9.1	35.7	76.3	-1665	1371	1471
1160	426.5	-157.4	-78.7	-78.7	-62.9	-22.0	19.0	56.1	-1801	1699	1332
1380	178.9	-157.4	-78.7	-78.7	-78.1	-43.7	-9.3	21.9	-2101	2346	1038
1580	13.6	-157.4	-78.7	-78.7	-78.7	-58.2	-28.2	-0.9	-2373	2840	780
1600	-0.7	-157.4	-78.7	-78.7	-78.7	-59.5	-29.8	-2.9	-2400	2886	754
1620	-14.6	-157.4	-78.7	-78.7	-78.7	-60.7	-31.4	-4.8	-2427	2932	728
1720	-76.5	-157.4	-78.7	-78.7	-78.7	-64.0	-37.4	-13.2	-2563	3147	599
1840	-136.2	-157.4	-78.7	-78.7	-78.7	-66.8	-43.0	-21.4	-2726	3387	444
1000000	-560.0	-157.4	-78.7	-78.7	-78.7	-78.7	-78.7	-78.7	-3227	4417	-325

axial force, the resistance bending moment has been calculated. Results for each reinforcing situation are shown in Fig. 10. In the example considered, i.e. a by-supported beam, axial forces are zero.

• *Comparison between the moment diagram and a selection of the definitive longitudinal reinforcement.*

Finally, internal bending moments (Fig. 8) and resistant moments were compared graphically for each type of reinforcement (Fig. 10). At this point, the aim was to check the recommended reinforcement and to optimise, when possible, the beam design. In the example used it can be concluded that reinforcement type 1 was not

**Figure 10.** Comparison between the internal bending moment diagram and the resistant moment diagram (for axial zero).

valid, because it could not support the internal bending moments. When considering reinforcements 2 and 3, no important differences were noticed. However, reinforcement 3, which has less steel weight than reinforcement 2, presented the best fit to the internal stresses diagram. It was also found that the section where both the resistant and the internal bending moment were best adjusted was located approximately at 3-3.5 m from the supports. This is not the central section because this section has the greatest concrete area, i.e. its resistance increases more than its internal stresses. Hence, the beam design could be optimised by trying to find the resistant moment diagram that was best adjusted to the internal bending moment diagram. Lightening of the central section of the beam, or an irregular beam design (boomerang kind, for instance) could be proposed.

Conclusions

When optimisation is the criterion to design and calculate structural elements, acceptable solutions are achieved by detailed and refined calculation methods. However, better procedures are required to optimise the results. For this reason, software tools that can help with the calculations, adapted to the requirements of each situation are essential.

The tool proposed here can be used to calculate all section types and to check all reinforcements of beams available on the market. It can, therefore, constitute a very helpful tool to those responsible for implementing projects in which this kind of product is used.

Introduction of the variable section height into the process is an important step forward because it allows a previously difficult issue to be solved. Moreover, the practical value of this tool is reflected by the good approximations obtained to real results.

The graphs that show the resistant bending moments give an idea of possible lightenings the resistant section could be submitted to, in the central area of the beam.

Thus, the possibility of reducing the section in this area can also be considered. This would be associated with different reinforcement locations, so the model of the beams could be optimized, helping to reduce both their weight and dimensions without reducing their potential resistance.

Acknowledgements

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