Annex: The Derivation of Rosenbaum Bounds

Let us assume that the probability of the treatment D for observation i is a function of the observed vector of covariates x_i and unobserved variable u_i. More precisely, $P(D_i = 1|x_i, u_i) = F(\beta x_i + \gamma u_i)$, where F is the logistic function and γ is the effect of the unobserved variable on the probability of treatment. When $\gamma = 0$, this means that the study is free of hidden bias and the selection into treatment is determined solely by x_i. When $\gamma \neq 0$, two observations, say *i* and *j*, which have the same covariates $x_i = x_j$, can have different probabilities of treatment if $u_i \neq u_j$. Since F is logistic, the odds of treatment for the two observations are $\frac{P_i}{1-P_i}$ and $\frac{P_j}{1-P_j}$, and the odds ratio is given by $\frac{P_j(1-P_i)}{P_i(1-P_j)} = \frac{e^{\beta x_i + \gamma u_i}}{e^{\beta x_j + \gamma u_j}} = e^{\gamma(u_i - u_j)}$. If the unobserved variable does not exert any influence ($i.e.,$ if $\gamma = 0$), or if $u_i = u_j$, then $e^{\gamma(u_i - u_j)} = 1$. Rosenbaum (2002) showed that the following bounds could be put on the odds ratio: $\frac{1}{\Gamma} = \frac{1}{e^{\gamma}} \leq \frac{P_j(1-P_i)}{P_i(1-P_j)} \leq e^{\gamma} = \Gamma$. Both observations have the same probability to be in treatment only if $\Gamma = e^{\gamma} = 1$. If for example $\Gamma = e^{\gamma} = 2$, that means that the probability that observation *i* receives treatment can be up to twice as high as the probability for observation *j*, regardless of the fact that probability should be the same for both units according to the observables, which is the result of hidden bias. This is how Rosenbaum bound Γ measures the extent of the hidden bias.