Annex A. Calculation of C_{SX}, C_{EX}, and α_{max}

Calculation of the starting point of rotation to exit the row (C_{SX})

The trajectory of the outsider point of the THF (point B, Fig. A.1), a cycloid function, is described by the following parametric functions:

$$x_B = C_{SX} + t \cdot v_a + R \cdot \sin(\omega \cdot t) \qquad (A.1)$$

$$y_B = Y_{IR} - R \cdot [1 - \cos(\omega \cdot t)] \qquad (A.2)$$

where:

C_{SX} = the x-coordinate of the RB centre in which the rotation starts skipping the plant; the condition to calculate C_{SX} is given by the tangency of the cycloid with the ZR circle.

Y_{IR} = sets how much the WT enters the row. In this work Y_{IR} = - r_t (the radius of the working tool)

R = overall rotation radius of the THF (Fig. A.1).

Starting from C_{SX}, the THF rotates by α_{max} to reach a position where the WT does not affect the RZ (in C_{EX} position).

After a rotation of an angle of α_{tan} ($\alpha_{tan} > \alpha_{max}$), in a time t_{tan}, the point B of the cycloid will be tangent to the RZ circle (Fig. A.1). In the meantime the C_{SX} point advances by a distance of $v_a \cdot t_{tan}$.

Referring to the triangle OCT, α_{tan} can be calculated by the following equation:

$$\alpha_{\tan} = a\cos\left(\frac{R - r_t}{R + r_r}\right) \qquad (A.3)$$

while t_{tan} will be:

$$t_{\tan} = \frac{\omega_o}{\alpha_{\tan}} \qquad (A.4)$$

Angular speed ω_o is not calculated, but it is given, depending on design needs. The abscissa of the C_{SXT}, the centre of the THF when B point is tangent to the RZ (T in Fig. A.1), is given by $T = -(R + r_r)\sin(\alpha_{tan})$; the starting point of rotation as in step 1 is in C_{SX}, that is backwards with respect to T by $v_a \cdot t_{tan}$. So the abscissa where the rotation starts skipping the plant will be:

$$C_{SX} = -[(R + r_r) \cdot \sin(\alpha_{\tan}) + v_a \cdot t_{\tan}] \qquad (A.5)$$

Calculation of the ending point of rotation to exit the row(C_{EX})

The rotation will end when the circle of WT is just outsidethe RZ as y-coordinate (the centre of rotation ofthe THF is in the C_{EX} position in Fig. A.1). The condition to save the RZ is given by the following:

$$r_r + r_t = -[R + r_r \cdot \cos(\alpha_{\tan})] \qquad (A.6)$$

From which we obtain:

$$\alpha_{\max} = a\cos\left(-\frac{r_r + 2r_t}{R}\right) \qquad (A.7)$$

And

$$t_{\max} = \frac{\omega_o}{\alpha_{\max}} \qquad (A.8)$$

Therefore C_{EX} will be:

$$C_{EX} = C_{SX} + v_a \cdot t_{\max} \qquad (A.9)$$

The angle α_{max} depends only on dimensional parameters (R, r_r, and r_t,), while C_{SX} depends also on v_a and ω_o.

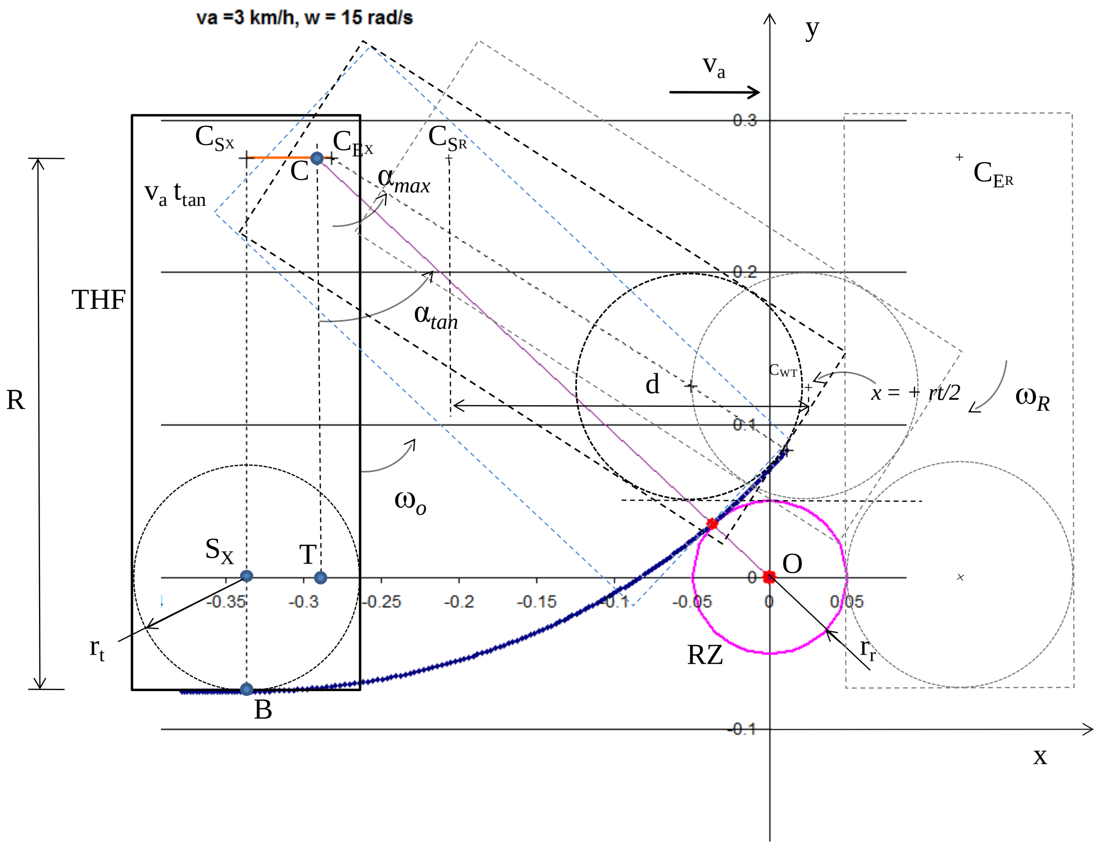

Figure A1. Sketch used to calculate the start/end rotation position to exit the row (respectively SX, EX) for FBTS-VR and FBTS-CR models (the exit procedure for both is the same). The sketch is used also to calculate the start/end rotation position to enter the row (respectively SR, ER) for the FBTS-VR model only.

Annex B. Calculation of the starting point (C_{SR}) of rotation to return in the row for FBTS-VR model.

Considering that translation goes from point C_{EX} to point C_{SR}, which corresponds to the position of C_{WT} at $+r_t/2$ with respect to Fig. A.1, C_{SR} can be calculated as:

$$C_{SR} = +\frac{r_r}{2} - d \qquad (B.1)$$

where:

$$d = (R - r_r) \cdot \sin(\alpha_{\max}) \qquad (B.2)$$

Annex C. Calculation of $x_{A\,min}$ and α_{min} for RPSS-HA model

Referring to Fig. 7, and taking into account the composed motion of rotation and translation, the coordinates of the point A as a function of time is given by the following parametric equations:

$$y_A = R - R^{'} \cdot \cos\left(\omega t + \beta\right) \qquad (C.1)$$

$$x_A = v_a \cdot t - R^{'} \cdot \sin\left(\omega t + \beta\right) \qquad (C.2)$$

where β is calculated as follows:

$$\beta = \arctan\left(\frac{r_t}{R}\right) \qquad (C.3)$$

And R' as:

$$R^{'} = \sqrt{R^2 + r_t^2} \qquad (C.4)$$

By setting to zero the derivative of Eqn. C.2, it is possible to obtain the time t_{min}, that is to say the time needed by A (Fig. 7) to reach the minimum value $x_{A\ min}$ (the maximum distance if considered in absolute terms):

$$t_{\min} = \frac{1}{\omega_{rot}}\left[\arccos\left(\frac{v_a}{\omega_{rot}\,R^{'}}\right) - \beta\right] \qquad (C.5)$$

To obtain $x_{A\,min}$ we can replace β and t_{min} into Eqn C.2 with Eqn C.3 and Eqn C.5, respectively:

$$x_{A\min} = \frac{v_a}{\omega_{rot}}\left[\arccos\left(\frac{v_a}{\omega_{rot}\,R^{'}}\right) - \arctan\left(\frac{r_t}{R}\right)\right] \; - R^{'} \cdot \sin\left(\arccos\left(\frac{v_a}{\omega_{rot}\,R^{'}}\right)\right) \qquad (C.6)$$

The angular position of the THF at $x_{A\,min}$ is given by the following equation:

$$\alpha_{\min} = \omega_{rot} \cdot t_{\min} = \arccos\left(\frac{v_a}{\omega_{rot}\,R^{'}}\right) - \beta \qquad (C.7)$$