

Applicability of the finite element method to analyze the stresses produced in concrete slabs over ground by tire loads of agricultural tractors

H. Malón, F. J. García-Ramos*, P. Hernández, M. Vidal and A. Boné

Escuela Politécnica Superior. Universidad de Zaragoza. Ctra. Cuarte, s/n. 22071 Huesca. Spain

Abstract

There are several methods to dimension concrete slabs due to vehicle loads, most of them based on Westergaard theory. These methods have been developed for industrial vehicles (cars, trucks and forklifts). Considering agricultural buildings one of the most used vehicles is the agricultural tractor whose characteristics (tires of great dimensions but with a reduced contact surface) are different to those of the industrial vehicles. The goal of this research was to analyze the applicability of the finite element method (FEM) to estimate the stresses generated on the concrete slabs considering the loads transmitted by agricultural tractors. To achieve this objective, the effect of the loads transmitted by the rear axle tires of three agricultural tractors has been considered. In parallel, the same study has been carried out using the Westergaard theory. As a preliminary step, to validate the FEM, a numerical analysis has been made to obtain the stresses generated on a concrete slab considering three forklifts. The numerical analysis results have been compared with those obtained by mean of validated methods (Portland Cement Association) and the classical theory of Westergaard. For each agricultural tractor, the actual geometry of the contact surface of tires has been measured on a concrete slab and discretized by the FEM. As a result of the research process developed, it is possible to conclude that the FEM is a valid tool to analyze the tensions generated by the loads transmitted by the tires of agricultural tractors on concrete floors supported on the ground.

Additional key words: agricultural tractor; concrete slab; FEM; tire.

Introduction

Concrete floors of agro-industrial buildings are commonly designed with slabs over the ground, which are reinforced for shrinkage and expansion owing to temperature changes. These slabs are designed to remain uncracked due to loads placed on the surface. Shrinkage cracking is controlled by a nominal or small amount of distributed reinforcement placed in the upper half of the slab. One of the most usual loads applied to the slabs are those derived from vehicular traffic (Beskou & Theodorakopoulos, 2011).

The American Concrete Institute suggests the following methods for selecting the thickness of the slab due to vehicle loads (ACI, 2010): the Portland Cement Association (PCA) method, the Wire Reinforcement Institute (WRI) method, and the Corps of Engineers

(COE) method. These methods are focused in loads applied by industrial vehicles: forklifts and distribution trucks with payload capacities as high as 310 kN. The payload and much of a truck's weight are generally carried by the wheels of the loaded axle. In this sense, the contact area of a single tire can be approximated by dividing the tire load by the tire pressure. However there are not specific methodologies to select the slab thickness for the case of agricultural vehicles as tractors which tire surfaces are not flat and consist of rubber pads of different height and geometry for a better contact during the field work (Sharma & Pandey, 1999). This fact has, as a consequence, a reduction in the tire contact area to a third part of that produced with a flat tire (Malón *et al.*, 2011). In this sense, the effects of the contact area between slab and tire must be studied to understand the load effects on the slab deformations and stresses.

* Corresponding author: fjavier@unizar.es
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Considering vehicle loads, variables affecting the thickness selection and design of slabs on grade include the following (ACI, 2010): maximum axle load, distance between loaded wheels, tire contact area, and load repetitions during service life. Considering heavy vehicles, damage on pavements is largely insensitive to axle spacing down to the limits dictated by current tire diameters (Gillespie, 1993).

Design methods are based on the use of charts and equations and have been developed based on the theories described originally by Westergaard (1926) who developed one of the first rigorous theories of structural behavior of rigid pavements. This theory considers a homogeneous, isotropic, and elastic slab resting on an ideal subgrade that exerts, at all points, a vertical reactive pressure proportional to the deflection of the slab. This is known as a Winkler subgrade. In this sense, many studies about the dynamic response of rigid pavement due to moving loads have been developed by modelling the pavement as a thin plate resting on Winkler foundation (Alvappillai *et al.*, 1992; Huang & Thambiratnam, 2002). The subgrade is assumed to act as a linear spring (Sawant, 2009) with a proportionality constant k with units of pressure per unit deformation. This is the constant now recognized as the coefficient of subgrade reaction, more commonly called the modulus of soil reaction or modulus of subgrade reaction.

On one hand, the analysis of the load capacity of concrete slabs can be estimated by experimental tests. Gaedicke *et al.* (2009), developed fatigue tests in concrete slabs applying peak loads of 90 kN to concrete slabs of 2,000 mm × 2,000 mm × 150 mm thick. Pavic & Reynolds (2003), carried out cracking test on a slab of 15,000 mm × 15,000 mm × 250 mm thick.

On the other hand, numerical methods, which are commonly identified with Finite Element Method (FEM), overcome the obstacles related with the complexities in experimental tests, geometric shapes and boundary conditions. Many studies have used FEM to analyze flexible pavement structure (Sebastian & McConnel, 2000; Goktepe *et al.*, 2006; Bailey *et al.*, 2008). As an example, Kim *et al.* (2007), employed two-dimensional (2-D) and three-dimensional (3-D) FEM to characterize the deformations of concrete pavements. Ioannides *et al.* (2006), had earlier published load-deformation results from 3-D finite element modeling of concrete slabs on an elastic foundation.

In this sense the FEM has been shown as a useful tool to estimate the stresses generated on the slabs of agro-industrial buildings due to vehicle loads

(Ferrer *et al.*, 2000; Sebastian *et al.*, 2000; Bailey *et al.*, 2008).

The goal of this study has been to analyze the stresses generated by agricultural tractor tires on concrete slabs over ground by using FEM as an alternative to traditional methods: the PCA method and the Westergaard theory. To achieve this goal, the transmitted load and the geometry of the contact area between tire and concrete have been studied specifically considering three agricultural tractors.

Material and methods

Theoretical model

Stresses generated by forklift and tractor tires on concrete slabs have been calculated. For this purpose, different techniques, such as the FEM (Kim *et al.*, 2007), the PCA method (ACI, 2010) and the Westergaard theory (Westergaard, 1926), have been used.

The steps carried out to obtain the slab size according to the Westergaard theory are described below. These equations are used to calculate the maximum concrete traction stress generated by a tire on the bottom surface of the slab.

The relative stiffness L is calculated according to Eq [1]:

$$L = \sqrt[4]{\frac{E \times h^3}{12 \times (1 - \mu^2) \times K}} \quad [1]$$

where E is the concrete Young modulus; μ is the concrete Poisson coefficient; h is the slab thickness; K is the foundation stiffness.

For loads inside a plate at a certain minimum distance from the edges, the Westergaard theory provides the maximum bending stress by means of the Eq [2]:

$$\sigma = \frac{3(1 + \mu)P}{2\pi h^2} \times \left(\ln \frac{L}{b} + 0.6159\right) \quad [2]$$

where P is the load of each tire, and b the equivalent radius of the circle on which the load is applied. The equivalent radius is obtained from the true radius a (Eq [3]), only when $a < 1.724h$, otherwise the true radius is used.

$$b = \sqrt{1.6 \cdot a^2 + h^2} - 0.675 \cdot h \quad [3]$$

The true radius of the circle should be obtained by measurements in situ, which is not a method commonly viable in the process of analysis of a tire track. For this reason the most widely technique used is to obtain this

information from the relationship between the load and the working pressure on the tire that is moving.

In the case in which the contact surface between the tire and the slab is considered as elliptical rather than circular, the critical stresses generated by a force in this area are calculated using the Eq [4]. In this equation c and d are the major semi-axis and minor semi-axis of the ellipse respectively:

$$\sigma = \frac{P}{h^2} \times \left\{ 0.275(1+\mu) \log_{10} \frac{E h^3}{K[(c+d)/2]^4} + 0.293(1-\mu) \frac{c-d}{c+d} \right\} [4]$$

Application of the FEM for sizing concrete slabs for forklifts

The first step in the research process developed is the application of the FEM in the dimensioning of the concrete slabs for forklifts. The results obtained by means of the FEM have been compared with the results of the PCA method and the Westergaard theory for the case of industrial vehicles. It must be considered that the PCA method, based on the use of charts for loads of industrial vehicles, is accepted as a reference method (ACI, 2010) for dimensioning concrete slabs.

In the study the loads of three forklifts have been considered: Still R 70-50 Diesel/GLP (small forklift load), Still R 70-80 Diesel (medium forklift load) and Yale GDP 100 DB Diesel (high forklift load). The slab thickness for each forklift has been calculated using the PCA method. In the application of this method the thickness of the concrete slab is obtained considering that the material works with the maximum traction stress. For concrete used in slabs, the allowable traction stress considered has been 1.665 N mm^{-2} . The thicknesses obtained using the PCA have been 219 mm, 265 mm and 280 mm for the small, medium and high forklift load, respectively.

The slab stresses have been calculated by means of the FEM and the Westergaard theory. In all the forklift load cases analyzed, the contact area between the tire of the forklift and the slab has been considered as circular.

In the numerical analysis by mean of the FEM, two types of slab models have been analyzed. The first type has been discretized with shell elements of four nodes (S4R) and the second type with volumetric elements (C3D8).

In the dimensioning of models the following has been considered: that the distance from the center of load application, the center of the tire forklift track, to the nearest edge of the slab must be greater than $1.75 \cdot L$ (Ioannides *et al.*, 1985), where L is the relative stiffness (Eq [1]).

Therefore six numerical models of slabs have been discretized, a shell type model and a volumetric type model for each of the forklift types. The main characteristics of the slab models are shown in Table 1.

The load cases calculated correspond to the force exerted on the slab by the two wheels of the axle of maximum load of the forklifts. The loads have been introduced as a downward vertical force of 50,420N (small forklift load), 70,921N (medium forklift load) and 92,517N (high forklift load) for each wheel of the forklifts analysed. These forces have been applied as distributed loads in circular areas, which correspond to the contact surface between the forklift tire and the slab. The contact areas of each forklift tire have been measured and their surfaces are shown in Table 1. The vertical forces applied on the circular areas are transmitted to the slab through contacts. The circular areas in which the load is applied have been discretized with shell elements of four nodes (S4R) calculated in all cases.

The materials used in the numerical analysis are concrete for all slabs, and rubber for the circular areas

Table 1. Characteristics of finite element models of slabs for forklift

Forklift model	Still R 70-50		Still R 70-80		Yale GDP 100 D8	
	Shell model	Volumetric model	Shell model	Volumetric model	Shell model	Volumetric model
Track width (mm)		1,210		1,447		1,570
Tire contact area (cm ²)		504.24		410.43		641.00
Relative stiffness (mm)		949.8		1,095.8		981.4
Slab length (mm)		4,550		5,287		5,010
Slab width (mm)		3,340		3,840		3,440
N° nodes	10,535	92,623	13,235	142,845	12,761	137,631
N° elements	10,612	81,592	13,300	128,752	12,820	123,952

Table 2. Mechanical properties of the materials used in the numerical analysis: concrete (EHE, 2008), forklifts rubber (data supplied by the Department of Mechanical Engineering of the University of Zaragoza and used in Carrera *et al.*, 2004) and tractor rubber (Flores *et al.*, 2010)

	Density (kg m ⁻³)	Young modulus (MPa)	Poisson ratio
Concrete	2,300	27,264	0.15
Forklifts rubber	1,000	100	0.47
Tractor rubber	1,153	203	0.47

in which the load is applied. The mechanical properties of these materials are shown in Table 2.

The boundary condition imposed in all load cases is an elastic foundation under slabs. This foundation reproduces the behaviour of the ground under slabs when loads are applied on them. To define the foundation behavior it is necessary to specify the foundation stiffness per unit area. In this study the foundation stiffness considered has been 0.02 MPa mm⁻¹ which can be assimilated to a soil of low quality, similar to wet clay. It has been selected a low foundation stiffness with the goal of obtaining conservative results (low values of the foundation stiffness require higher slab thickness). This boundary condition has been applied on the bottom surface of the slabs analyzed.

At the same time that the FEM analysis has been carried out, the slabs have also been calculated using the Westergaard theory. The numerical calculation results obtained using the Westergaard theory have been used to check the correlation of the numerical results obtained by means of the FEM with the Westergaard theory results. This correlation allows for validating the FEM for dimensioning of slabs.

Application of the FEM for sizing concrete slabs for agricultural tractors

In the sizing process of concrete slabs for agricultural tractors the loads transmitted by the tires of the rear axle of three tractors (Massey Ferguson 277, Fiat 80-90S and New Holland G 170) have been considered. For each type of tractor, different values of wheel load and tire pressure have been considered. The load transmitted by each wheel has been measured in real conditions. Table 3 shows the different load cases analyzed.

For each load case the stresses of the concrete slab have been obtained by means of the Westergaard theory

Table 3. Load cases analyzed for each agricultural tractor

Tractor	Rear tire	Rear tire pressure (bar)	Rear wheel load (N)	Load case
Massey Ferguson 277	14.9-28 6 pr	0.7	8,389	1
		1.1	8,389	2
		1.1	11,524	3
Fiat 80-90 S	16.9/14-34 8pr	0.7	13,441	4
		1	13,441	5
		1.3	13,441	6
New Holland G 17	620/70R42	1.5	24,892	7
		1.5	30,772	8
		1.9	24,892	9

and the FEM. An elliptical contact surface between the tire and the concrete slab has been considered in the Westergaard theory (Hallonborg, 1996).

For the finite elements analysis, the contact surface between the tire of each tractor and the slab has been measured by experimental test in field (Lyasco, 1994). These measurements have yielded the exact geometry of the rubber blocks of the tractor tires, in which the contact between tire and slab is generated.

A dipping technique has been applied to obtain these geometries. This technique consists of dipping the tire surface in water, and then moves the tractor over the slab to mark the exact tire track on the floor. These results obtained by means of the dipping technique are the exact geometries of the tire track of each load case analysed in this study. An example of the tire track obtained is shown in the Fig. 1.

Analogously to the process of using the FEM to sizing slabs for forklifts, two finite element types of slab have been modelled for the sizing of the slabs for agricultural tractors. The first type has been discretized

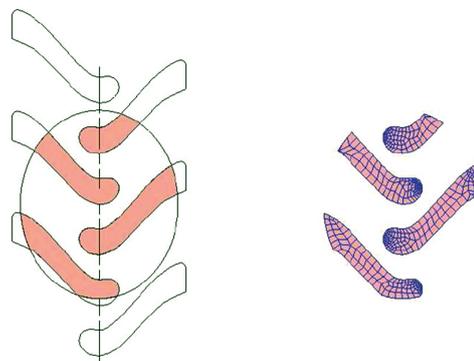


Figure 1. Example of the tire track obtained and its finite elements model (FEM) of the Massey Ferguson 277, with a tire pressure of 1.1 bar (load case 2).

with shell elements of four nodes (S4R) and the second type with volumetric elements (C3D8). In this analysis, all models have a thickness of 200 mm, due to the fact that the PCA method has not been developed for tractor tires. A thickness of 200 mm is an usual value of concrete slabs of agro-industrial buildings.

It has been considered that the distance from the center of load application, the center of the tire tractor, to the nearest edge of the slab must be greater than $3*L$. This value has been increased with respect to the forklift loads analysis due to the increase in size of the tractor tire tracks.

Geometries of the tire track are discretized with shell elements of four nodes (S4R), and a FEM of tire track has been discretized for each load case analyzed. Fig. 2 shows the top view of the FEM for the load case 4, in which the slab has been discretized with volumetric elements. The number of nodes and discretized elements for the analysis of slabs for tractor tires are shown in Table 4.

The calculated load cases correspond to the force exerted on the slab by the wheels of the rear axle of the tractors analyzed. These load cases are shown in Table 3. The loads have been applied as a download vertical force for each load case calculated. The application areas of forces correspond to the geometry of the rubber blocks of the agricultural tires obtained by the dipping technique exposed previously.

The boundary conditions imposed are the same as the boundary conditions of numerical analysis developed in concrete slabs for forklifts. An elastic foundation has been applied to the bottom surface of the slabs, which has a foundation stiffness of 0.02 MPa mm^{-1} .

The materials used in the numerical analysis are concrete and rubber. Concrete is applied in all slabs.

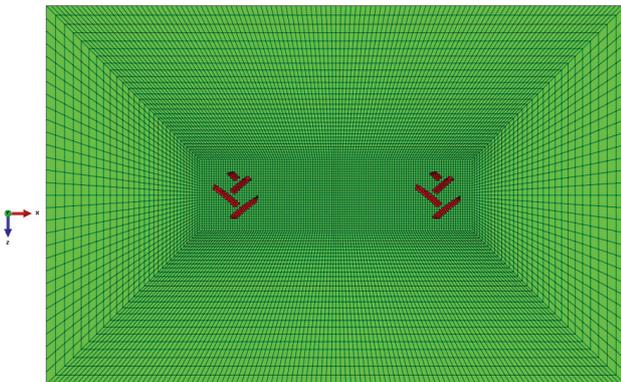


Figure 2. Top view of the volumetric finite elements model for the FIAT 80-90 S with a tire pressure of 0.7 bar (load case 4).

Table 4. Number of nodes and elements of the finite element models of slabs for tractors

Load case	Shell model		Volumetric model	
	Number nodes	Number elements	Number nodes	Number elements
1	10,563	10,230	48,727	38,480
2	10,509	10,188	48,673	38,448
3	10,965	10,570	49,129	38,830
4	13,355	12,958	62,903	49,678
5	13,365	12,964	62,913	49,684
6	12,929	12,636	62,477	49,684
7	18,623	18,196	123,581	104,796
8	18,359	17,980	123,317	104,580
9	18,795	18,342	123,753	104,942

Rubber is used for the application areas of forces, which corresponds to the geometry of the rubber blocks of the agricultural tires. The mechanical properties of these materials are shown in Table 2.

Results and discussion

Application of the FEM for sizing concrete slabs for forklift

This section compares the numerical results obtained by the PCA method and the Westergaard theory with the results obtained by means of the FEM. To do this, the maximum bending stresses on the bottom surface of the slab have been compared in the study.

As an example Fig. 3 shows the maximum bending stress on the bottom surface of the slab obtained by the

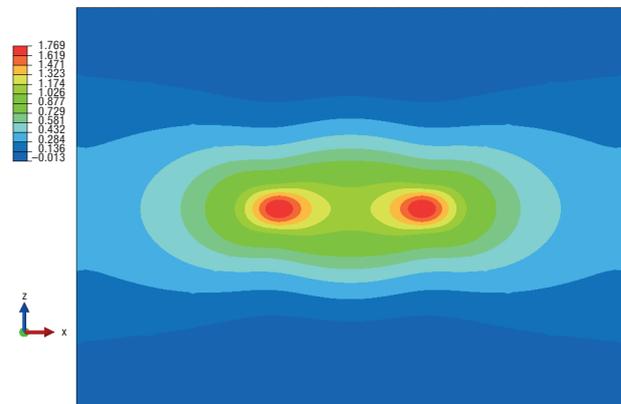


Figure 3. Bending stresses (MPa) on the bottom surface of the slab obtained by means of the FEM with a shell slab model and the load case of the forklift Still R 70-50 Diesel/LPG.

Table 5. Maximum bending stresses obtained by mean of the PCA method and the FEM for forklifts

Type of forklift	PCA method	FEM, shell model		FEM, volumetric model	
	Maximum bending stress (MPa)	Maximum bending stress (MPa)	Error (%)	Maximum bending stress (MPa)	Error (%)
Still R 70-50	1.665	1.768	6,19	1.617	-2.88
Still R 70-80	1.665	1.816	9,07	1.662	-0.18
Yale Gdp 100 D8	1.665	1.832	10,03	1.676	0.66

Table 6. Maximum bending stresses obtained by mean of the Westergaard theory and the FEM for the slab models for forklifts

Type of forklift	Maximum bending stress-direction 1					Maximum bending stress-direction 2				
	Westergaard (MPa)	FEM (MPa)		Error (%)		Westergaard (MPa)	FEM (MPa)		Error (%)	
		Shell	Volumetric	Shell	Volumetric		Shell	Volumetric	Shell	Volumetric
Still R 70-50	1.450	1.520	1.386	4.83	-4.41	1.771	1.768	1.617	-0.17	-8.70
Still R 70-80	1.466	1.584	1.44	8.05	-1.77	1.770	1.816	1.662	2.60	-6.10
Yale Gdp 100 D8	1.503	1.584	1.441	5.39	-4.13	1.814	1.832	1.676	0.99	-7.61

FEM for the load case of the forklift Still R 70-50 Diesel/LPG, in which a shell slab model has been used.

The results obtained by means of the PCA method and the FEM are shown in Table 5. Table 6 shows the maximum bending stress in the two main directions of the slabs bottom surface, which have been obtained by mean of the FEM and the Westergaard theory.

The results obtained by means of the PCA method and the FEM show low differences which are lower in volumetric slab models. In all the cases of volumetric slab models, the difference of the results of the FEM with respect to the PCA method are less than 2.9% while in the shell models the difference are less than 10.03%. These errors were concordant with those obtained by Gaedicke *et al.* (2012), who obtained a difference of 1%-5% between the simulated and experimental peak load capacity considering 150 mm depth slabs.

According to results of Table 5, the FEM can be applied as a valid technique in the sizing of concrete slabs because of its similarity with the PCA method. The maximum error obtained is 10.03%. These errors allow validating the FEM in the sizing of concrete slabs with respect to the PCA Method.

Concerning the Westergaard theory, the correlation of the results obtained by means of the FEM and the numerical theory also show low differences. In this case two results have been analyzed: the maximum bending stress in the two main directions of the bottom surface of the slabs. The better correlation of bending

stress results obtained by the Westergaard theory and the FEM are shown in the shell slab models due to the fact that Westergaard theory and the finite elements calculation with shells are based on the thin plate theory. This result agrees with that obtained by Ferrer *et al.* (2000), who obtained a great correlation between the Westergaard theory and a shell FEM considering forklifts loads.

These low differences between results of the PCA method, Westward theory, and the FEM allow using the FEM as a technique to analyze the stresses in concrete slabs and to dimension their size.

Application of the FEM for sizing concrete slabs for agricultural tractor

Once the application of the FEM for the analysis of concrete slabs was finished, a study based on this technique has been started for sizing concrete slabs for agricultural tractors.

To do this, the maximum bending stresses on the bottom surface of the slab obtained by means of the Westergaard theory and the FEM have been compared in the study. In this case it is not possible to analyze the results of PCA method due to the fact that this method is not developed for tractor tires. The results obtained of shell and volumetric slab models are shown in Table 7.

Table 7. Maximum bending stresses obtained by mean of the Westergaard theory and the FEM for slab models for agricultural tractors

Type of tractor	Load case	Maximum bending stress-direction 1					Maximum bending stress-direction 2				
		Westergaard (MPa)	FEM (MPa)		Error (%)		Westergaard (MPa)	FEM (MPa)		Error (%)	
			Shell	Volumetric	Shell	Volumetric		Shell	Volumetric	Shell	Volumetric
Massey Ferguson 277	1	1.188	1.250	1.012	5.22	-14.81	1.352	1.377	1.117	1.85	-17.38
	2	1.268	1.325	1.070	4.50	-15.62	1.431	1.446	1.174	1.05	-17.96
	3	1.529	1.626	1.321	6.34	-13.60	1.731	1.805	1.461	4.27	-15.60
Fiat 80-90 S	4	1.433	1.574	1.244	9.84	-13.19	1.489	1.574	1.238	5.71	-16.86
	5	1.483	1.635	1.293	10.25	-12.81	1.565	1.668	1.313	6.58	-16.10
	6	1.597	1.755	1.389	9.89	-13.02	1.738	1.819	1.433	4.66	-17.55
New Holland G 17	7	1.404	1.519	1.282	8.19	-8.69	1.563	1.632	1.377	4.41	-11.90
	8	1.520	1.615	1.361	6.25	-9.99	1.671	1.738	1.467	4.01	-12.21
	9	1.608	1.731	1.467	7.65	-8.77	1.801	1.870	1.588	3.83	-11.83

As in the analysis of the forklifts, the results obtained by the FEM with shell slab models show lower differences with the Westergaard theory ones than the results obtained from the volumetric models by the FEM. This is due to the fact that both techniques are based on the thin plate theory (Ferrer *et al.*, 2000). However, the results shown in Tables 5, 6 and 7 (oversizing of the slab thickness using the thin plate theory with respect to the PCA method) support the use of the FEM volumetric elements to analyze slab stresses.

In all cases studied, the error has increased with respect to the forklift cases. This increase could be due to the geometry of the contact areas of the tires with the slabs applied in the FEMs. The Westergaard theory considers that the contact surface between the tire and the slab occurs in one uniform surface but with tractor tires this is not really the case. In the case of the tractor, the contact area between the tire and the slab occurs only on the rubber blocks of the tires. For this reason all results obtained by means of the FEM with slab shell models are greater than the Westergaard theory results.

In the volumetric FEMs of slabs, the maximum bending stresses obtained are lower than the results obtained by the Westergaard theory.

As an example of the obtained results, Figs. 4 and 5a show the maximum bending stresses on the bottom surface of the slab calculated by the FEM with shell and volumetric models respectively for the load case 5. Fig. 5b shows the bending stresses in the section of the slab considering the load case 5. The evolution of the bending stresses is concordant with that obtained by Ferrer *et al.* (2000) and Gaedicke *et al.* (2012) considering loads on concrete slabs over ground.

In addition to the bending stresses the numerical analysis by means of the FEM has yielded the minimum thickness of the slabs for the three types of tractors analysed. These thicknesses are 78 mm for the Massey Ferguson 277, 84 mm for FIAT 80-90S, and 116 mm for the G 170 New Holland. In all load cases, the thicknesses obtained are lower than the thickness currently used in agro-industrial buildings.

In conclusion, the FEM can be applied successfully to obtain stresses and strains in slabs due to loads of agricultural tractors. In this sense, the application of the FEM correlates to the theoretical results of the Westergaard theory for both the shell slab models and the volumetric models.

The results obtained by calculation techniques based on the thin plate theory show bending stresses higher than other techniques. Therefore the slab thickness

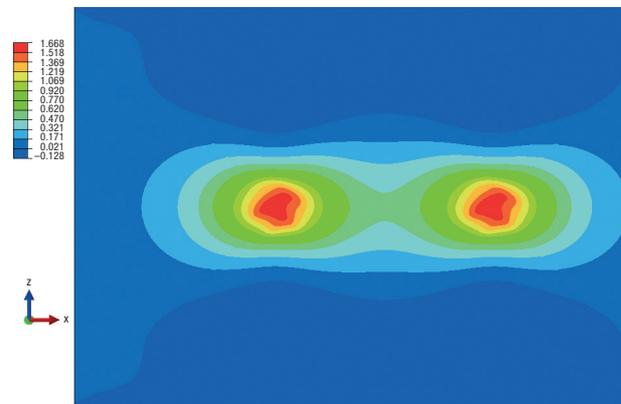


Figure 4. Bending stresses (MPa) on the bottom surface of the slab obtained by means of the FEM with a shell slab model for the tractor Fiat with a tire pressure of 1 bar (load case 5).

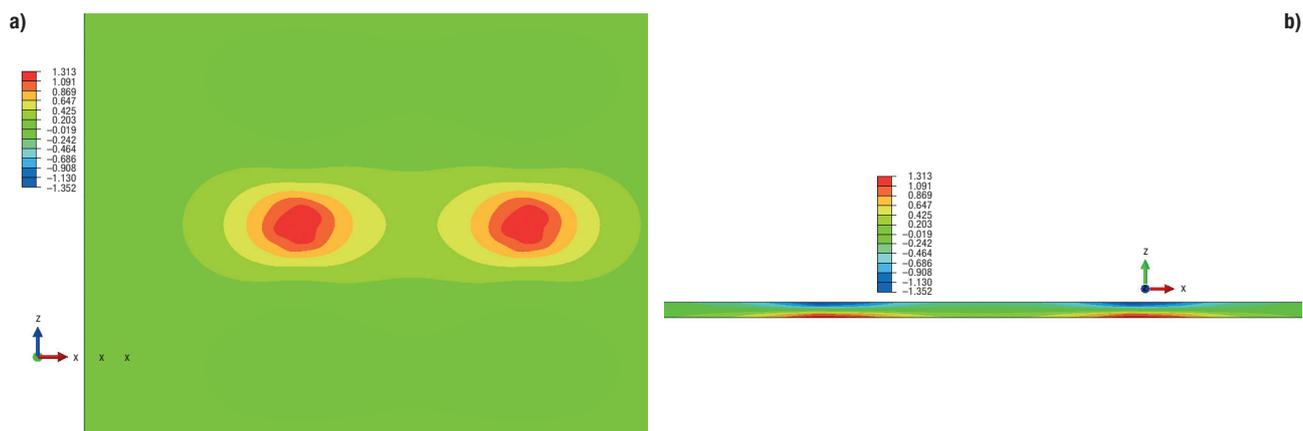


Figure 5. Bending stresses (MPa) in the bottom surface (a) and in the central section (b) of the slab obtained by means of the FEM with a volumetric slab model for the tractor FIAT with a tire pressure of 1 bar (load case 5).

obtained by the techniques based on the thin plate theory is oversized with respect to the PCA.

The results obtained by means of the FEM with volumetric elements show that for tires of agricultural tractors, and considering a foundation stiffness of 0.02MPa/mm, the minimum slab thickness is less than 200mm which is concordant with the thickness used in current agroindustrial buildings.

In this sense, the application of the FEM is a useful tool that allows design engineers technically justify the sizing of concrete slabs, considering agricultural tractor loads.

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