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Assessment of methods for obtaining rainfall intensity-duration-frequency ratios for various geographical areas

A. J. Zapata-Sierra[1]*, F. Manzano-Agugliaro[1] and J. L. Ayuso-Muñoz[2]

[1] *Departamento de Ingeniería Rural. Escuela Politécnica Superior, Universidad de Almería (UAL), La Cañada de San Urbano, s/n. 04120 Almería, Spain.*

[2] *Departamento de Ingeniería Rural. Escuela Técnica Superior de Ingenieros Agrónomos y de Montes, Universidad de Córdoba (UCO), Campus de Rabanales. Córdoba, Spain.*

Abstract

Several procedures for obtaining rainfall intensity-duration-frequency (IDF) relationships for storm durations ranging from 10 min to 24 h and for several return periods were analyzed. The data recorded for the period 1966–1997 are from five different locations in southern Spain: one mountain, two inland, and two littoral stations. We analyzed the original equations of Wenzel, Bell, Chen and Témez, and other ones modified from the last two. The first step was to determine the parameters of each equation for each geographical location. Subsequently, the coefficient of variation (CV) and index of agreement (i_a) of each equation were calculated in order to compare their estimations for rainfall durations of less than 2 h and less than 24 h. The results show that there is not one equation which is best suited for all the geographical areas or rainfall durations. The reference method, Wenzel's equation, is the best for rainfall periods of less than 24 h, but only for the littoral and inland geographical areas. If Wenzel's equation is not taken into account, the modified Témez equation proved to be the best at generating series for mountain areas and for the two rainfall periods studied. Therefore, in a regional study of rainfall durations of less than 24 h, where only 24 h data are available, and with littoral, inland and mountain areas, the modified Témez equation is strongly recommended.

Additional key words: geographic area, IDF, littoral, mountain, rainfall.

Resumen

Evaluación de métodos de obtención de las relaciones intensidad-duración-frecuencia de lluvia para distintas zonas geográficas

En el presente trabajo se analizan diferentes métodos de obtener las curvas de intensidad duración y frecuencia de lluvia para distintos periodos de retorno y duración de la lluvia. Para ello se han empleado los datos de 5 estaciones pluviométricas en el sur de España, siendo dos de ellas estaciones litorales, otras dos interiores y una de montaña. Los registros pluviométricos analizados son desde 1966 a 1997. Se han analizado las ecuaciones de Wenzel, Bell, Chen y Témez, y otras modificadas a partir de las dos últimas. El primer paso fue determinar los parámetros de todas las ecuaciones para cada una de las localizaciones geográficas. Posteriormente se calcularon los coeficientes de variación (CV) e índice de agregación (i_a) de cada ecuación para poder comparar las estimaciones realizadas por cada ecuación en dos duraciones de lluvia: menor de 2 h y menor de 24 h. Se observó que no existe ninguna ecuación que sea la mejor para todas las localizaciones geográficas y las dos duraciones. El método de referencia, ecuación de Wenzel, es el mejor método para duraciones de menos de 24 horas en zonas de litoral y de interior. Si la ecuación de Wenzel no se tiene en cuenta, la ecuación de Témez modificada resulta ser la mejor para las zonas de montaña y los dos periodos estudiados. Para estudios regionales con zonas de litoral, montaña e interior, disponiendo sólo de datos de 24 horas, y duraciones de menos de 24 horas, la ecuación de Témez modificada es la más aconsejable.

Palabras clave adicionales: IDF, litoral, montaña, lluvia, zona geográfica.

* Corresponding author: ajzapata@ual.es
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Abbreviations used: CV (coefficient of variation), i_a (index of agreement), IDF (intensity–duration–frequency), T (return period).

Introduction

Estimates of mean rainfall intensity, for a given duration and selected return periods, are of interest to hydrologists, hydraulic engineers, and many other users for hydrological planning and design (Yu *et al.,* 2004), for hydrologic risk analysis and design in channeling flash-flood watercourses, levees, drainage structures or urban storm-drain systems or soil water erosion studies.

In this regard, the rainfall intensity–duration–frequency (IDF) ratios for a location, expressed mathematically, allow the mean rainfall intensity to be calculated, designed for a given return period and a wide range of rainfall durations. These ratios are usually represented graphically by means of what are known as IDF curves. Wenzel (1982) and Chow *et al.* (1988) describe the equations that express IDF ratios based on an area's rainfall records, when rainfall intensity data over different time periods is available, and are considered to be the reference methods when calculating IDF curves. In many areas, however, fragments of the rainfall record may be missing (Svenson *et al.*, 2007) or are not available in much detail, being limited to the daily rainfall measured with totalizing rainfall gauges. In such cases, it is possible to obtain generalized IDF ratios from isopluvial maps published for large geographical regions (Bell, 1969; Chen, 1983; Froehlich, 1993, 1995; Durrans and Kirby, 2004).

Thus, it is possible to obtain IDF ratios in Spain based on a regionalization study of the ratio between the rainfall mean intensity for 1 h and for 24 h, using the daily rainfall data for a geographical location (Témez, 1987). Although the results obtained by this method are only approximate, they are still useful.

The aim of this paper is to compare different existing methods for obtaining IDF expressions. Therefore, for the existing methods and two modified ones, we applied and analyzed the various procedures for obtaining IDF ratios, based on the two approaches described above: the reference method (Chow *et al.*, 1988) and other approximate methods.

Material and methods

Rainfall data

To assess the different methods for obtaining IDF ratios in coastal and inland areas, five first-class rainfall stations in southern Spain were chosen: for coastal areas, Málaga (36.69º N, 4.49º E) and Almería (36.84º N, 2.37º E) airports; for inland areas, Córdoba (37.84º N, 4.85º E) and Sevilla (37.42ºN, 5.89º E) airports; for mountain areas, the Lanjarón Forestry Experimentation Station (Granada) (36.92º N, 3.49º E) on the southern slopes of the Sierra Nevada mountains (see Figure 1 for further location details). The series of rainfall records analyzed at these stations were: 1985–97, 1980–97, 1982–97, 1980–97, and 1966–97, respectively. The length of these data series is quite common for this type of studies. At all the observatories, the series of maximum annual rainfall were available for the following intervals: 10, 20 and 30 min, and 1, 2, 3, 6, 12 and 24 h.

In Figure 2 we observe a data sample from the Almería Station, where the high variability of rainfall intensity for the period 1980–1997 can be observed.

Intensity–duration–frequency analysis

At each station, frequency analysis was carried out using the maximum annual rainfall for each of the rainfall durations selected, by fitting each series to a Gumbel distribution (Kite, 1977), modified according to equation [1], using the maximum-likelihood method.

$$P(X \leq x) = \exp\{-\exp[-\alpha(x-\beta)]\} \quad [1]$$

Where $\alpha = \frac{\pi}{\sqrt{6}\ S}$ and S is standard deviation of the sample; $\beta = \bar{x} - \frac{0.5772}{\alpha}$ and $\bar{x}$ is average of the sample; and X is the random variable.

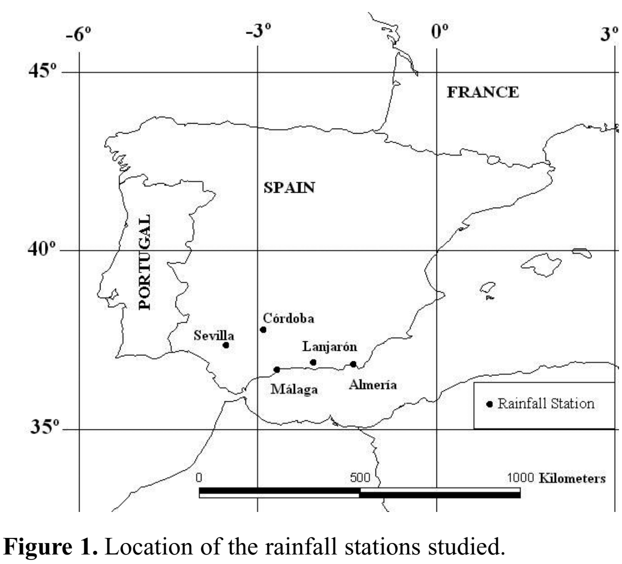

Figure 1. Location of the rainfall stations studied.

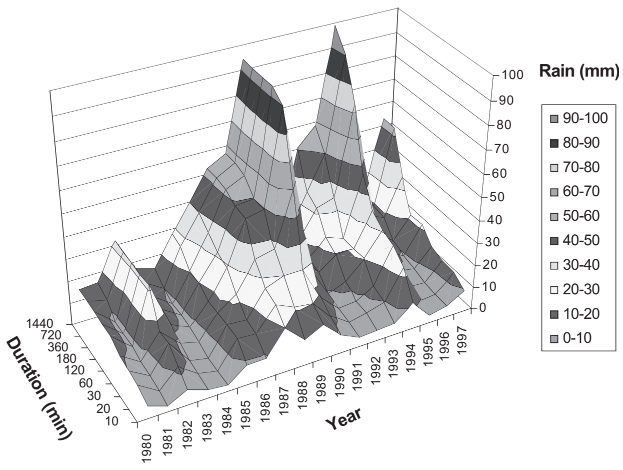

Figure 2. Data sample from Almería station.

For the return periods T=2, 5, 10, 25, 50 and 100 years, the rainfall-height values, R_t^T , were obtained for each rainfall duration considered, t, and the corresponding intensities, r_t^T.

For a further analysis of fitting of the Gumbel distribution, a coefficient of correlation between observed and estimated data was calculated for all the data set. The results show that this coefficient of correlation was always higher than 0.94 for all the data set.

Intensity–duration–frequency relationships

IDF relationship can be described mathematically by means of various expressions (Wenzel, 1982). The most common one, which groups the various intensity–duration curves for the various return periods in a single formula, is equation [2], which is applicable to locations with observatories keeping records for rainfall durations between 10 min and 24 h.

$$r_t^T = \frac{a\,T^b}{t^c + d} \qquad [2]$$

where r_t^T is the mean intensity (mm h^{-1}) for the duration t (min) and the return period T (years), and a, b, c and d are parameters to be determined by fitting.

In cases where only 24 h rainfall data is available, regional rainfall characterization studies are carried out analyzing the ratios between short-lasting rainfall and rainfall over 1 h and/or 24 h (Bell, 1969; Chen, 1983; Froehlich, 1993 and 1995). Using isohyetal rainfall maps for large regions of the USA, Chen (1983) obtained a ratio between the rainfall height for 1 h and 24 h, regardless of the return period, (R_1^T/R_{24}^T), that varies very little according to the geographical location, ranging between values of 0.1 and 0.6, with an average value of 0.4. Bell (1969) studied the empirical relationships between rainfall lasting between 5 min and 2 h and 1 h, proposing the ratios expressed in equations [3] and [4]:

$$R_t^T = \left(0.54\,t^{0.25} - 0.5\right)\left\{0.35 ln \left[ln \left(\frac{T}{T-1}\right)\right]^{-1} + 0.76\right\} R_1^2 \qquad [3]$$

$$R_t^T = \left(0.54\,t^{0.25} - 0.5\right)\left\{0.21 ln \left[ln \left(\frac{T}{T-1}\right)\right]^{-1} + 0.52\right\} R_1^{10} \qquad [4]$$

where t is the rainfall duration (min), T is the return period (years), and R_t^T is the rainfall with duration t and return period T; thus, R_1^{10} is the rainfall for 1 h and 10 years, while R_1^2 is the rainfall for 1 h and 2 years. Chen (1983), using three rainfall heights: 1 h and 10 years (R_1^{10}), 24 h and 10 years (R_{24}^{10}), and 1 h and 100 years (R_1^{100}), obtained generalized IDF ratios. Alternatively, by beginning with the intensity-duration ratio expressed in equation [5],

$$r = \frac{a}{(t+b)^c}, \qquad [5]$$

Chen established equation [6] as the intensity-duration ratio, regardless of the return period:

$$\frac{r_t^T}{r_1^T} = \frac{a_1}{(t+b_1)^{c_1}}, \qquad [6]$$

where r_t^T is the mean intensity (mm h^{-1}) of rainfall with duration t (min or h) and a return period of T (years); thus, r_1^T is the rainfall mean intensity for 1 h and T years. The expression [6] can be written as in equation [7]:

$$r_t^T = \frac{a_1 r_1^T}{(t+b_1)^{c_1}}, \qquad [7]$$

which is identical to equation [5] with $a=a_1r_1^T$; $b=b_1$ and $c=c_1$. The fitting parameters a_1, b_1 and c_1 can be obtained from the known rainfall data from a given station by using optimization techniques and the least squares method.

Chen proposed the following ratio, expressed in equation [8]:

$$\frac{r_t^T}{r_t^{10}} = \frac{R_t^T}{R_t^{10}} = log\left(10^{2-x}\;T_p^{x-1}\right), \qquad [8]$$

where T_p is the return period for a series of partial duration, and x is the R_t^{100}/R_t^{10} ratio. The value of T_p is related to the return period of the annual maximum series T, by the following expression (Kite, 1977):

$$T_p = \left[ln \left(\frac{T}{T-1} \right) \right]^{-1} \qquad [9]$$

When equation [8] is built into equation [7], we need to specify equation [8] for 1 h, and we get:

$$\frac{r_1^t}{r_1^{10}} = \frac{R_1^T}{R_1^{10}} = log \left(10^{2-x} \; T_p^{x-1} \right) \qquad [10]$$

where x now has the value R_1^{100}/R_1^{10}. By replacing equation [10] in equation [7] and taking into account [9], we obtain:

$$r_t^T = \frac{a_1 r_1^{10} log \left\{ 10^{2-x} \left[ln \left(\frac{T}{T-1} \right) \right]^{-(x-1)} \right\}}{(t+b_1)^{c_1}} \qquad [11]$$

which are the relationships proposed by Chen.

If in equation [7] we take the 24 h rainfall mean intensity as the reference instead of the 1 h mean intensity, equation [7] becomes:

$$r_t^T = \frac{a_{24} r_{24}^T}{(t+b_{24})^{c_{24}}} \qquad [12]$$

By comparing equations [7] and [12] we obtain the ratios between parameters $a_{24}=a_1 r_1^T/r_{24}^T$; $b_{24}=b_1$ and $c_{24}=c_1$. Expression [11] becomes:

$$r_t^T = \frac{a_{24} r_{24}^{10} log \left\{ 10^{2-x} \left[ln \left(\frac{T}{T-1} \right) \right]^{-(x-1)} \right\}}{(t+b_{24})^{c_{24}}} \qquad [13]$$

where $x=R_{24}^{100}/R_{24}^{10}$. These equations allow us to obtain the IDF ratios from 24 h rainfall data.

Since the IDF curves present a degree of similarity, they can be represented by a single adimensional law, expressing the intensities as percentages of a mean intensity associated with a given reference duration. If we take 24 h as the reference duration–since it is available at all the observatories–the adimensional law for a family of curves is thus:

$$\frac{r_t^T}{r_{24}^T} = \phi(t) \qquad [14]$$

Témez (1987) characterizes this law by means of the ratio r_1^T/r_{24}^T, such that equation [14] becomes

$$r_t^T = r_{24}^T \left(\frac{r_1^t}{r_{24}^T} \right)^{\frac{28^{0,1}-t^{0,1}}{28^{0,1}-1}} \qquad [15]$$

where t is rainfall duration in h.

The r_1^T/r_{24}^T ratio is independent of the return period, it only depends on the geographical location of the area. The value of this parameter is regionalized for Spain (Témez, 1987).

Equation [15] can be modified to be expressed as:

$$r_t^T = r_{24}^T \left(\frac{r_1^t}{r_{24}^T} \right)^{\alpha_1 + \beta_1 \ln t}, \qquad [16]$$

where the coefficients a_1 and b_1 can be determined by using optimization techniques based on the observed intensity data.

Results and discussion

To analyze and assess the validity and precision of the methods described for obtaining IDF ratios and determining the most suitable procedure for each area, we proceeded to apply them to the five stations described.

First, Wenzel's equation [2] was fitted to the rainfall-intensity data obtained from each station by means of frequency analysis ("observed data"), obtaining values for the parameters *a, b, c,* and *d* expressed in Table 1. We then proceeded to estimate the rainfall-intensity values for the different durations and return periods by applying Bell's equations [3] and [4]; Chen's equation [11] with the coefficients a_1, b_1 and c_1, determined by using optimization techniques; Chen's modified equation [13], applying the coefficients a_{24}, b_{24} and c_{24}; and Témez's equation [15] and modified equation [16], optimizing the parameters *a* and *b*. The values for the parameters in equations [7], [12], and [16] determined by optimization are shown in Table 1. The values of parameter r_1^T/r_{24}^T used in Témez's equations [15] and [16] are also shown in Table 1.

In order to compare the estimates made by each procedure, we defined a coefficient of variation as the ratio between the square root of the mean squared error and the mean of the rainfall values observed,

$$CV = \frac{\sqrt{\frac{\sum_{i=1}^{n} (x_{i_0} - x_{i_c})^2}{n}}}{\frac{\sum x_{i_0}}{n}} \qquad [17]$$

where x_{i0} are the values obtained for the rainfall heights of the different rainfall durations and return periods, x_{ic} are the rainfall heights calculated for the different durations (10 min to 24 h) and return periods (2 to 100 years), and n is the number of rainfall data employed for each equation (*i.e.* n=54, 9 durations and 6 return period in equations [2], [11], [13], [15] and [16]).

Table 1. Values calculated for the parameters of the following equations: Wenzel [2], Chen [11], Chen modified [13], Témez [15] and Témez modified [16], for rainfall data in the stations studied

Equation	Parameter	Málaga	Almería	Lanjarón	Córdoba	Sevilla
Wenzel	a	699.93	812.47	154.28	218.36	1999.08
[2]	b	0.2385	0.2460	0.2356	0.1641	0.1637
	c	0.7330	0.8495	0.5946	0.5735	0.9896
	d	3.3052	12.3976	−0.4561	0.4316	23.7475
Chen	a_1	18.81	32.41	11.91	8.71	11.730
[11]	b_1	5.78	16.96	−1.08	0.46	0.33
	c_1	0.69	0.79	0.61	0.55	0.57
Chen modif.	a_{24}	208.09	381.61	83.91	72.74	99.58
[13]	b_{24}	5.78	16.96	−1.08	0.46	0.33
	c_{24}	0.69	0.79	0.61	0.55	0.57
Témez modif.	α_1	−0.312	−0.301	−0.314	−0.306	−0.312
[16]	β_1	2.308	2.227	2.289	2.233	2.278
Témez	r_1^T/r_{24}^T	10.738	11.677	7.084	8.333	9.130
[15 and 16]						

These analyses have also been done with index of agreement (i_a) (Willmott, 1982),

$$i_a = 1 - \frac{\left[\sum_{i=1}^{n} (x_{i0} - x_{ic})^2\right]}{\sum_{i=1}^{n} \left(|x_{i0}^*| + |x_{ic}^*|\right)^2} \qquad [18]$$

where

$|x_{i0}^*| = |x_{i0} - \overline{x_{i0}}|$, $|x_{ic}^*| = |x_{ic} - \overline{x_{i0}}|$, and $\overline{x_{i0}}$ is the average of obtained data.

Bell's equations [3] and [4] are valid only for estimating rainfall between 10 min and 2 h, so they were only compared with the values obtained by the other methods for this same interval.

An analysis was carried out for the five rainfall stations and two data sets: the last 10 years and the full data set. The coefficient of variation (CV) for each rainfall station and each expression in both data sets was computed, and the average CV for each equation calculated in order to compare the CV of both data sets.

The CV of the analyzed equations for rain durations of less than 2 h between both data sets show the same trend (Figure 3A), although the full data set has smaller CV for all equations. Figure 3B shows the CV for the analyzed equations for rain durations of less than 24 h between both data sets. Again, the same trend is observed and the full data set has smaller CV for all equations as well. All the equations have CV variations lower than 15% for rainfall durations of less than 2 h, see Figure 3A. CV for Wenzel's equation is the least sensitive to rainfall duration in both cases. The modified Chen's equation [13] does not improve the original Chen's equation [11]. On the other hand, the modified Témez equation [16] is better than the original one [15]

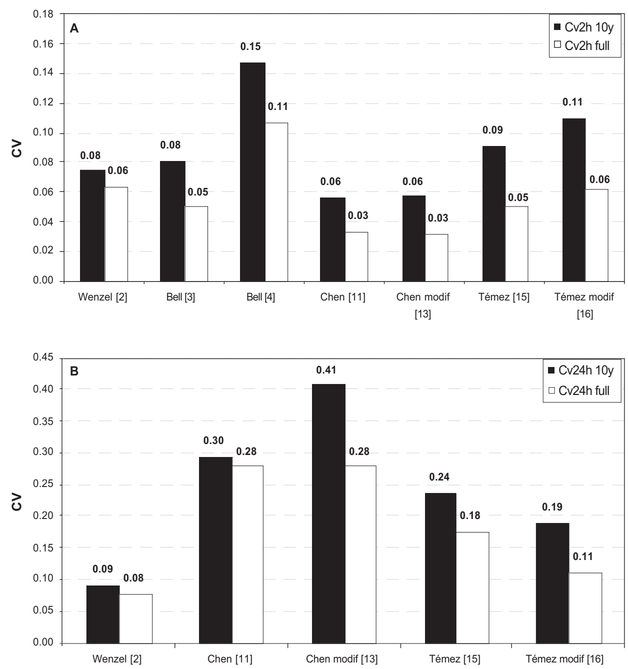

Figure 3. Coefficient of variation (CV) values obtained with the different equations for rainfall durations of less than 2 h (A) and less than 24 h (B) and for different lengths of data sets (10 years and full series).

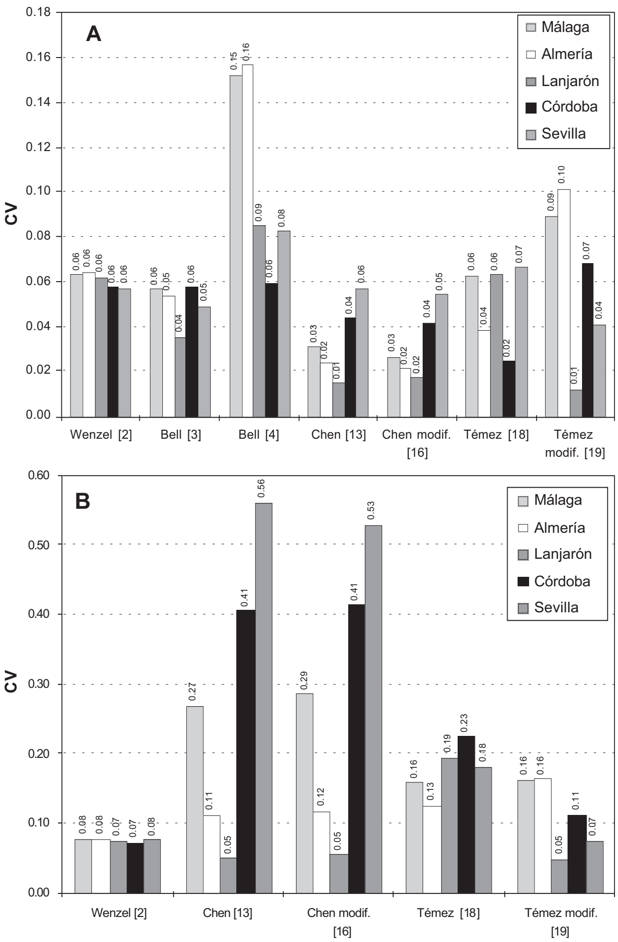

Figure 4. Coefficient of variation (CV) values obtained with the different equations for rainfall durations of less than 2 h (A) and 24 h (B) at the different stations studied.

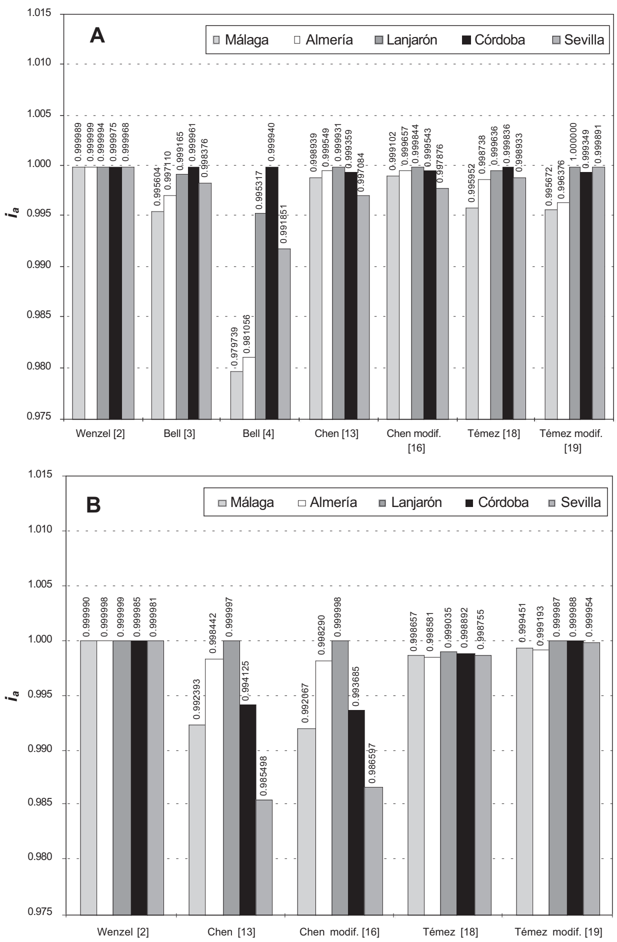

Figure 5. Index of agreement (i_a) values obtained with the different equations for rainfall durations of rain less than 2 h (A) and 24 h (B) at the different stations studied.

in short series (35% of CV) as is full ones (70% of CV), for durations of less than 24 h.

In a second step, the differences between the five rainfall stations were analyzed. Figure 4A shows the coefficient of variation (CV) and Figure 5A shows the index of agreement (i_a) obtained with the different methods for rainfall durations of less than 2 h. Figure 4B shows the CV and Figure 5B shows the index of agreement (i_a) obtained with the various expressions to generate the complete data set from 10 min to 24 h for rainfall heights for the different return periods. Wenzel's equation shows a similar behavior for all the geographical areas and Bell's equation [4] shows the most variable one.

In order to compare the different geographical areas, the average for inland and littoral stations was calculated. Table 2 shows the three best equations for both estimators (CV and i_a) for rain durations of less than 2 h and less than 24 h in all geographical areas. For rain durations of less than 2 h, "Témez modified" [16] is the best equation for both estimators in mountain areas. If the Wenzel equation [2] is not taken into account (it is the reference equation), then the "Chen modified" [13] is the best equation for littoral areas.

For rain durations of less than 24 h, the Wenzel equation [2] is the best for inland and littoral areas (for both the CV and i_a estimators). If equation [2] is not taken into account, "Témez modified" [16] is the best equation for inland and mountain areas, while the Témez [15] or the modified Témez [16] equations are the best for littoral areas. Therefore, "Témez modified" [16] is one of the three best equations for both estimators and all geographical areas. Also, if Wenzel's equation [2] is not taken into account, "Témez modified" [16] is the

Table 2. Summarized results obtained from littoral, inland and mountain geographical areas for rainfall durations of less than 2 or 24 h. CV (coefficient of variation), i_a (index of agreement)

	Littoral		**Inland**		**Mountain**	
	CV	i_a	**CV**	i_a	**CV**	i_a
2 h						
1st	Chen Mod. [13]	Wenzel [2]	Témez [15]	Wenzel [2]	Témez Mod. [16]	Témez Mod. [16]
2nd	Chen [11]	Chen Mod. [13]	Chen Mod. [13]	Témez Mod. [16]	Chen [11]	Wenzel [2]
3rd	Témez [15]	Chen [11]	Chen [11]	Témez [15]	Chen Mod. [13]	Chen [11]
24 h						
1st	Wenzel [2]	Wenzel [2]	Wenzel [2]	Wenzel [2]	Témez Mod. [16]	Wenzel [2]
2nd	Témez [15]	Témez Mod. [16]	Témez Mod. [16]	Témez Mod. [16]	Chen [11]	Témez Mod. [16]
3rd	Témez Mod. [16]	Témez [15]	Témez [15]	Témez [15]	Chen Mod. [13]	Témez [15]

best for mountain areas for both rainfall durations (< 2 h and < 24 h).

Conclusions

By comparing the various equations existing in the literature with the ones developed for this paper it was found that there is not one equation which best suits all the geographical areas or rainfall durations.

The reference method, Wenzel's equation, it is the best for rainfall periods of less than 24 h, but only for littoral and inland geographical areas.

If Wenzel's equation is not taken into account, the modified Témez equation proved to be the best at generating series for the two rainfall periods studied in mountain areas.

In conclusion, for regional studies of rainfall durations of less than 24 h, where only 24 h data are available, and with littoral, inland and mountain areas, the modified Témez equation is strongly recommended.

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