

Supplementary Table 1. Copula data and maximum pseudo-likelihood

Let the pairs of ranks $(R_{11}, R_{21}), \dots, (R_{1i}, R_{2i}), \dots, (R_{1N}, R_{2N})$, where R_{1i} stands for the rank of observation X_{1i} in the random vector X_1 and R_{2i} stands for the rank of observation X_{2i} in the random vector X_2). The copula data are derived by normalizing the ranks by a factor of $1/(N+1)$. The *empirical copula* is defined as

$$C_N(u_1, u_2) = \frac{\sum_{i=1}^N 1(\frac{R_{1i}}{N+1} \leq u_1, \frac{R_{2i}}{N+1} \leq u_2)}{N} \quad [\text{A.1}]$$

with domain $(0,1)^2$; $1(\cdot)$ is an indicator function. The method of maximum pseudo-likelihood maximizes the rank-based, log-likelihood function

$$l(\theta) = \sum_{i=1}^N \ln \{c_\theta(\frac{R_{1i}}{N+1}, \frac{R_{2i}}{N+1})\} \quad [\text{A.2}]$$

where c_θ is the PDF of a copula with parameter vector θ (e.g. Genest *et al.*, 1995; Genest & Favre, 2007).

Note that [A.2] is derived by replacing the unknown marginal distribution functions, F_1 and F_2 , in the classical log-likelihood function $l(\theta) = \sum_{i=1}^N \ln [c_\theta \{F_1(X_{1i}), F_2(X_{2i})\}]$ by

their empirical counterparts $F_{1N}(u_1) = \frac{\sum_{i=1}^N 1(X_{1i} \leq u_1)}{N+1}$ and $F_{2N}(u_2) = \frac{\sum_{i=1}^N 1(X_{2i} \leq u_2)}{N+1}$; θ is the parameter vector (equal to θ or ρ for the one-parameter copulas and to (θ_1, θ_2) for the two-parameter copulas, in accordance with Table 1).